The Coda of Bruckner's Fourth Symphony: A Mathematical Conglomeration

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Abstract. This paper describes the author's idiosyncratic listening experience of the coda to the Fourth Symphony of Anton Bruckner (1824-1896) as performed by the Romanian conductor Sergiu Celibidache. It employs mathematical analogues and aims to provide insight on how personal perception of structural phenomena affect the listening experience.

Keywords: Bruckner, coda, sequence, monotone, lim sup, lim inf, convergent, divergent, Shepard

1 Introduction

Josef Anton Bruckner (1894-1896), the Austrian composer known primarily for his symphonies and masses, has acquired a reputation today as a simple-minded and rustic musician whose works are often dismissed as stodgy and disjoint. Much of this image has to do with performance practice and the notion that Bruckner's music was only fit for Austrian audiences for a long time, but the efforts of conductors and scholars in the last several decades have helped bring Bruckner's music to a wider audience.

From a musical standpoint, Bruckner's symphonies are emblematic of the final stage of Austo-German romanticism: they are harmonically daring but formally conservative, longer than all other symphonies before his time, and highly polyphonic in style. Bruckner's background as an organist is sometimes evident in his trademark style of orchestration that employs brass and strings in novel ways. Furthermore, the surface level of Bruckner's music is often highly repetitive, which perhaps accounts for the dismissal of his music as simple-minded. On the other side of the spectrum, many have championed Bruckner as a musical genius: the German philosopher Ernst Bloch wrote of Bruckner's symphonies that "this music seems indeed part of a divine system of mathematics."

My paper explores my own listening experience to a particular recording of Bruckner's Fourth Symphony, focusing on the coda of the symphony. My intent is not to write an analytical monograph with tools like harmonic or Schenkerian analysis but rather to elucidate the elements of the music that are manifest to me and highlight them with appropriate analogues.

2 Analytical Overview

In the recording excerpt I have chosen, Sergiu Celibidache conducts the Munich Philharmonic Orchestra in February 1989 at a performance of Bruckner's Fourth Symphony in the Wiener Musikverein. The seventy-six-year-old Celibidache had already acquired a reputation of being a highly idiosyncratic Bruckner conductor, the primary reason being his massively slowed-down tempi.

The coda to the finale, which is comprised of sixty-four measures plus a quarter note, typically takes two to three minutes in recordings, but Celibidache's glacial tempi draw it out to nearly five minutes. As a consequence of the slower tempo, new and unique sonorities become manifest to me which I have attempted to explain here.

Due to Bruckner's tendency to revise his works repeatedly, there exist several different versions of the Fourth Symphony, and Celibidache conducts the 1881 edition as edited and published by musicologist Robert Haas in 1936. Structurally, the coda functions to recapitulate, unite, and resolve the primary themes of the symphony. A full-scale formal analysis of the coda, while worthy, is beyond the scope of this paper, so a truncated version will suffice. In lieu of a textual explanation, I have made a concise chart detailing what I hear (and see in the score) on the next page. This information is necessary to understand my interpretation of the coda. In my subsequent explanation of my listening, I will focus on the elements that are most striking to me, elements which are not necessarily always in the foreground of the music.

Measures	1-12	13-28	29-40	41-56	57-64
Instrumentation	French horn, oboe, clarinet, strings ostinato (***ppp***)	French horn and trombone, strings ostinato (***pp***)	full orchestra (≺)	full orchestra (≺)	full orchestra (***fff***)
Foreground	three-whole-note theme outlining I - I - ♭VI	chorale (mostly i, $ii^{\varnothing\,4}_{\;2}$, $iv^{(6)}$)	three-whole-note theme, crescendoing	ascent	E ♭ major peroration
Ostinato (only first violin pitches notated here)	♩ -triplet ♩ -triplet $\|\!:\overline{E\flat - F - E\flat}\;\overline{F - E\flat - F}:\!\|$			recursive ascent	♩ -triplet ♩ -triplet $\|\!:\overline{C\flat - B\flat - C\flat}\;\overline{B\flat - E\flat - B\flat}:\!\|$

Fig. 1. Basic Structural Analysis of the Coda

3 Mathematical Analogues

3.1 Ostinati

Celibidache's unconventional conducting is perhaps most effective in highlighting the firm stability of the string ostinato: in other recordings, I always perceive the very same ostinato as flowing and dynamic whereas here it strikes me as a rigid foundation. The pitches of the ostinato figure in the first forty bars of the coda are reminiscent of a type of sequence studied in mathematical analysis: the **non-monotone sequence**. It is an infinitely long list of elements (typically numbers) in which successive elements are not uniformly greater or less than their preceding elements. A **monotone sequence**, mathematically stated, is a sequence $\{a_n\}$ where either

$$a_i + 1 \geq a_i \text{ or } a_i + 1 \leq a_i \text{ for every } i \geq 1,$$

so a *non-monotone sequence* is simply one that does not have a uniform order relation between its elements as described above. The **limit superior**, or **lim sup**, is the value of the highest element reached or tended to by a non-monotone sequence, and the **limit inferior**, or **lim inf**, is the value of the lowest element reached or tended to by a non-monotone sequence. The diagram below shows the intuitive definitions of the lim sup and lim inf.

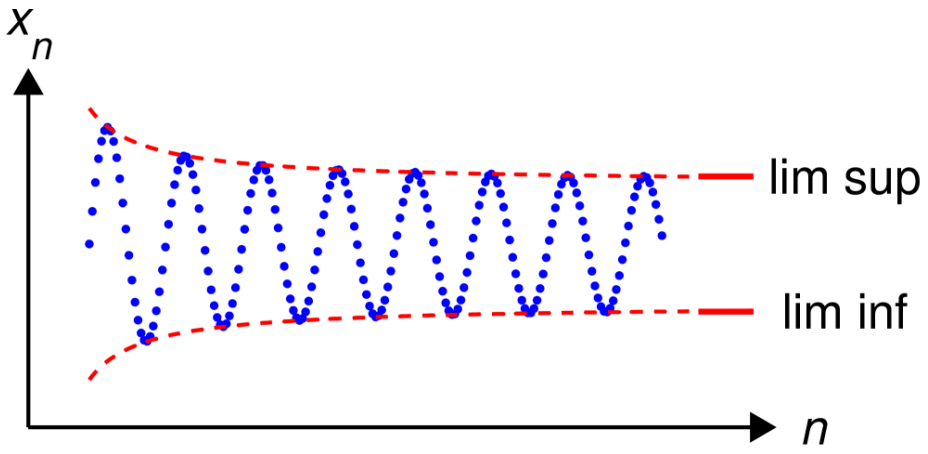

Fig. 2. Visual intuition of the limit superior (limsup) and imit inferior (liminf)

I hear Bruckner's ostinato as a non-monotone sequence as such, with a lim sup of F and and a lim inf of E♭. A sequence is **convergent** if it tends to one value as the elements go off to infinity. The convergence of Bruckner's sequence is yet to be determined as the ostinato seems to go on endlessly, especially at

Celibidache's tempo. Stated in terms of the lim sup and lim inf, a sequence converges if and only if the lim sup and lim inf become equal, as in the diagram below. The value to which they converge is known as the limit of a sequence. Consequently, only convergent sequences have limits.

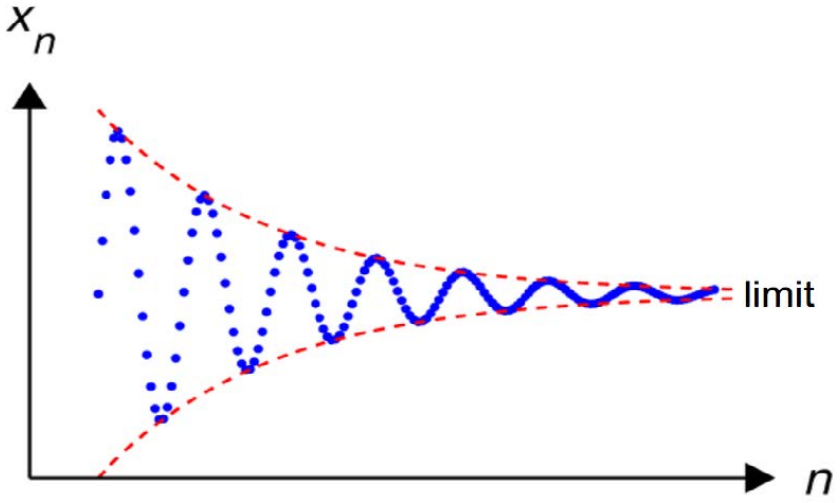

Fig. 3. A convergent sequence and its limit

As the symphony's tonal center is E♭, it would seem that the goal of the coda would be to force the lim sup of the ostinato sequence (F) down to converge with the E♭. In the segment that follows the triple iteration of the three-whole-note theme, the brass play a chorale that hovers around the tonic of E♭, refraining from any perfect cadences but instead frequently using $\text{ii}^{\text{ø}4}_{2}$ and iv chords to counterbalance the consonant i chords (the absence of a I chord except for at the very beginning of the chorale is curious). While the ostinato figure continues without pause in the strings, I hear this brass chorale as an extension of the non-monotone sequence, and in fact a struggle to push it toward convergence. The $\text{ii}^{\text{ø}4}_{2}$ chords are built on the root of F, corresponding to the F's in the strings' sequence, indicating a force pushing the sequence away from E♭. The i chords, built on E♭ but in the minor species instead of the expected major, come closer to establishing the convergence of the sequence, but the foreign G♭'s prevent them from doing so completely. In my interpretation, I visualize this strife as a narrowing of the gap between in the lim inf and the lim sup, i.e., the span of the extrema of the sequence becomes truncated. In actuality, this does not occur, since the E♭–F ostinato has remained unchanged, but my mind hears the E♭ minor triads as closer in proximity to the desired "limit" of E♭ major.

After the brass chorale, the three-whole-note theme is reiterated thrice again as in the beginning of the coda, this time increasing in volume with each iteration. Another added element is the trumpet line, insistently playing a triplet figure in two octaves of E♭. Both the increasing dynamic and the added trumpet line intensify the struggle for convergence. However, the significance of this passage is trivial in comparison to the following passage, hence its short treatment here. Just as the third iteration of the three-whole-note theme is about to finish, the expected third chord of ♭VI is replaced with the root position Neapolitan, ♭II. Another jarring change is the string ostinato, which, having played the same figure for forty seemingly interminable measures, has now shifted upward to the following figure (again, only the first violin's pitches are indicated below as they are most prominent in my listening):

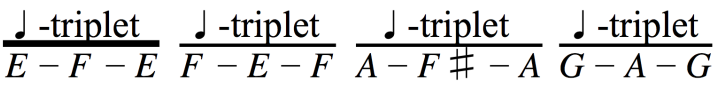

Fig. 4. The first violins' pitches beginning in measure 41

3.2 Ascent and Recursive Sequences

The string ostinato continues to ascend stepwise, preserving the contour of the four-triplet cell above. Before the rest of the orchestra takes my attention, I have time to formulate an intuitive framework for processing this figuration. The mathematical analogue of a recursively defined sequence, one that depends on and is defined by itself, springs to mind. The Fibonacci sequence, which begins with $\{1, 1, ...\}$ and is constructed further by letting each successive element equal the sum of the previous two elements, is the most common example of a recursive sequence. Bruckner's triplet figuration reminds me of a recursive sequence because of the nature of the melodic ascent: it is not predictable in chromatic steps but only in locally relevant diatonic steps. Aurally, the sequence, especially when heard atop the brass harmonies, appears to follow a rigorously predictable pattern of ascent, but a closer look at the first six bars reveals a discrepancy between the span of the lower neighboring tone and tonicized note across the first two tonicizations. The pattern is therefore subtly predictable and unpredictable alike: predictable if one analytically determines the recursive rule and unpredictable if one only relies on aural perception, as it was in my case. Figure 5 on the following page graphs the pitch discrepancies over time.

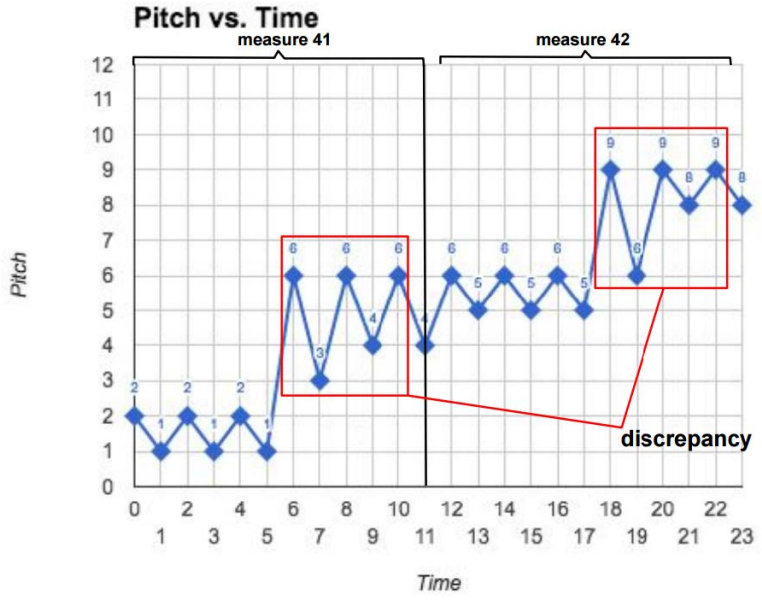

Fig. 5. Pitches versus time in mm. 41-42 of the coda. Note that movable pitch integer notation has been employed here: 0 = E♭, 1 = E, 2 = F,. . . ,12 = E♭

I have little time to ponder more on the melodic figuration as the brass harmonies in the foreground begin to occupy the foreground of the texture. In order to better understand my interpretation of this passage, the following harmonic analysis will likely be useful. As per standard notation, capitals denote major triads and lower case letters and numerals denote minor triads. In a feat of compositional virtuosity, every triad is in root position.

$$\begin{array}{cccccc} [\text{vi} — \text{V} — \text{I}] & [\text{V} — \text{I}] & [\text{V} — \text{I}] & [\text{V} — \text{I}] & [\text{V} — \text{iv} — \flat\text{III} — \frac{\flat\text{II}}{4\ bars} — \mathbf{I}] \\ \text{a} \to \text{G} \to \text{C} \to \text{B} \to \text{F}\flat \to \text{E}\flat \to \text{a}\flat \to \text{G}\flat \to \text{C}\flat \to \text{B}\flat \to \text{a}\flat \to \text{G}\flat \to \text{F}\flat \to \mathbf{E}\flat \end{array}$$

Fig. 6. Harmonic analysis of the "ascent" theme, mm. 41-56 of the coda

My ear for harmony is not so advanced that I can envision the above tonal plan just by listening, but there are certain qualities that do indeed strike me simply through listening. The most evident is the alternating dominant–tonic function of the chords, creating a sense of predictably shifting stability. This is not only a result of the brass, however: the sense of constantly shifting stability is further strengthened by the strings' ostinato sequence. On the tonic harmonies, the melodic triplets adumbrate the tonic note with upper neighboring tones, but on the dominant harmonies, the triplets stretch the span of this adumbration

with the inclusion of a lower neighboring tone. (This span expansion is a direct contrast to the span compression of the brass chorale earlier.) I hear this as a strengthening of the gravitational pull of the dominant toward the subsequent tonic. Although the entire passage leading up to the tonic peroration is gradually crescendoing, Bruckner's trick of including lower neighboring tones only on the dominant harmonies causes me to perceive the dominant harmonies as dynamically stronger (that is, in intensity of volume) than the tonic harmonies. This is a peculiar but nonetheless unshakeable aural illusion.

3.3 Ascent and the Shepard Scale

The second major interpretive takeaway I get from this passage is ascent, both melodic and harmonic (hence my concise name for the passage in the structural map of the coda). However, the feeling of ascent is not quite corroborated by the music itself: while the first violins are ascending along the pattern of the triplet ostinato, the lower brass and certain woodwind groups are actually *descending*. What, then, causes the sensorial perception of uniform ascent? I do not have the answer to this question, as my only evidence is my own perception, but I liken the effect of this passage to that of a Shepard scale. A Shepard tone, named after cognitive scientist Roger Shepard, consists of superimposed sine waves separated by octaves, and a Shepard scale is generated by moving the base pitch of the tone upward or downward. Resultantly, the Shepard scale produces the auditory illusion of a tone ascending or descending infinitely. The parallel is not exact to my perception of the ascent passage in Bruckner's coda, but it is common in the sense that I hear continual ascent in Bruckner's passage despite the presence of descent while an ascending Shepard scale typically gives the impression of continual ascent despite a circuitous order of pitches.

3.4 The Refunctioned Neapolitan

The third and final perceptive image I divine from the ascent passage is the immense gravity of the final root position Neapolitan chord, held for four seemingly eternal bars before breaking forth to the tonic peroration. Not only do I find it oddly natural that the Neapolitan gives way directly to the tonic (a motion not at all natural in traditional tonal harmony), I find it extremely compelling. The gravitation I intuit here is nearly in the literal sense as opposed to the gravity I hear in dominant-function chords: the flattened supertonic, being as close as possible to the tonic in pitch, is weighted down so heavily by the full and ever-crescendoing orchestra that I feel it must eventually break under its own force and come to rest on the stable ground of the tonic.

3.5 Peroration: Closure?

The eight-bar tonic peroration closing the coda is extremely gratifying as a result of the tremendous gravitational force preceding it, and it is here where the mental image of the non-monotone sequence returns to the foreground of my listening. (It had not disappeared during the ascent passage, but the harmonic function had diverted my focus temporarily.) In the music itself, the ***fff*** first violins are screaming out this triplet figuration,

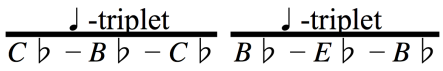

Fig. 7. Triplet figurations in the first violins in the final peroration, mm. 57-64

in which the upper neighboring tone to the dominant (namely, the C♭) refuses to settle completely, preserving the sense of shifting stability pervading the ascent passage. However, the rest of the orchestra, likewise playing ***fff***, could not be declaring E♭ major more forcefully. Yet the insistent C♭'s of the violins refuse to be drowned out. While the convergence of the non-monotone sequence would seem almost inexorable at this point, I hear this peroration as defiance. Even the entire orchestra cannot succeed in making the sequence converge: while the stray F has disappeared, the C♭ has taken its place. When the unadorned E♭ major chord finally arrives as the sole quarter note ending the symphony, it is gone before I can appreciate the true resolution. It leaves me wondering if convergence can ever be achieved. Even the rigorous mathematical definition of convergence incorporates a concept that is handwavy and unintuitive at best: that of infinity. If a sequence appears to tend to a certain value as the elements go off to infinity, how can one truly know if the sequence won't misbehave (i.e. diverge) at some point too far away to see? Convergence only works with sequences and functions defined either absolutely or recursively by a rule, but in the realm of music, surely a recursive rule cannot be trusted over artistic conception. My own aural afterimage of the ending of the Fourth Symphony is invaded with C♭, the foreign upper neighbor tone that prevents me from feeling true resolution in spite of the tonic chord.

4 Conclusion

Over the course of writing this paper and in my repeated listenings to Bruckner's coda, I have become increasingly aware of the dynamic nature of my interpretation. Some features are more consistent, but other nuances, like the precise articulation patterns of the various instrument groups or the subtle changes in tempo, shift in and out of focus on different listenings. This effect is not limited to Celibidache's recording but indeed seems to be more pronounced with it, perhaps because his generous tempi allow for greater interpretive flexibility on part of the musicians. To me, it is this fundamentally dynamic nature of aural perception that troubles the innate sacrosanctity of traditional analyses: if indeed listening is so individualistic, can we ever achieve a true musical analysis?

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